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During the project period, Ph.D. student Amites Dasgupta has been investigating "Fractional Brownian Motion: its Properties and Applications to Stochastic Integration"

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RESEARCH IN STOCHASTIC PROCESSES AND THEIR APPLICATIONS

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FINAL REPORT

March 31, 1997

Research in Stochastic Processes

G. Kallianpur, P.I.

Final Report

1. Since Professor Stamatis Cambanis passed away in April 1995, I have been guiding his student, Mr. Amites Dasgupta. This is the only work undertaken by me under the grant.

   Dasgupta has just completed his Ph.D. thesis and the final report consists of a brief summary of his research. His final Ph.D. exam is scheduled for June 25, 1997.

2. Research done by Amites Dasgupta: Fractional Brownian Motion: its Properties and Applications to Stochastic Integration.

During the project period I have been investigating " Fractional Brownian Motion: its Properties and Applications to Stochastic Integration". Some technical details are given below.

Normalized fractional Brownian motion (FBM), denoted $B_H(t)$ on $[0,T]$ is a Gaussian process with the following covariance

$$EB_H(t)B_H(s) = (1/2)\{|t|^{2H} + |s|^{2H} - |t-s|^{2H}\}, t, s \in [0,T], 0 < H < 1.$$

For $1/2 < H < 1$, $B_H(t)$ has been used as a model for physical phenomena exhibiting long range dependence, see e.g. Mandelbrot and Van Ness [7]. In practical problems one works with finite differences

$$\Delta B_H(t_k) = B_H(t_{k+1}) - B_H(t_k), 0 = t_0 < t_1 < \cdots < t_{n+1} = T.$$

This naturally leads to the question of integration with respect to FBM. We refer to Barton and Vincent Poor [1] for a problem in signal detection requiring such theories. However it is difficult to provide a general integration theory w.r.t. FBM because it is not a semimartingale unless $H = 1/2$. Our work examines how presence of dependent increments affects applications of fractional Brownian motion with special emphasis on stochastic integration.

We first provide a theory of integration of nonrandom functions of one variable with respect to FBM. This enables us to define an analogue of the White Noise measure for FBM and we call it the Fractional White Noise measure. This measure helps to define the derivative of FBM as a random functional in the Schwarz space of distributions. We also examine in what sense increments of FBM can be considered as a differential of the process in the context of smooth approximations as mentioned by Mandelbrot and Van Ness [7].

The effect of dependence in the increments stands out prominently in our theory of multiple integrals w.r.t. FBM. Let $0 = t_0 < t_1 < \cdots < t_{n+1} = T$ be a partition of $[0,T]$ and $1_{\Delta_i}(.)$ denote the indicator function of $[t_i, t_{i+1})$. Following Ito [3] we define the multiple integral of the elementary function

$$F(x_1, \cdots, x_p) = \sum_{i_1, \cdots, i_p} a_{i_1 \cdots i_p} 1_{\Delta_{i_1}}(x_1) \cdots 1_{\Delta_{i_p}}(x_p), x_1, \cdots, x_p \in [0,T],$$

with respect to FBM by

$$I_p(F) = \sum_{i_1,\cdots,i_p} a_{i_1\cdots i_p} \Delta B_H(t_{i_1}) \cdots \Delta B_H(t_{i_p}).$$

We derive a formula for the second moment of $I_p(F)$ in terms of the reproducing kernel Hilbert space of FBM. This second moment can be bounded by the L^2 norm of F w.r.t. a certain finite measure μ_p on $[0,T]^p$. With the help of this bound the definition of the multiple integral is extended in mean square sense to larger classes of nonrandom functions on $[0,T]^p$. We also find that when doing multiple integration of nonrandom functions w.r.t. FBM presence of diagonals does not cause any difference in the definition (as it does in the case of Wiener process). This helps us to prove the Stratonovich type formula (in the mean square sense)

$$\int_0^T [B_H(u)]^{p-1} dB_H(u) = \frac{1}{p}[B_H(T)]^p,$$

where p is a positive integer, which is quite different from the formulas obtained in the presence of Wiener process.

We briefly study the chaos decomposition of the multiple integrals and get representation of chaos components in terms of multiple integrals. The chaos decomposition shows that even and odd order multiple integrals are orthogonal to one another but in general even (resp. odd) order multiple integrals are not orthogonal among themselves unlike the multiple Wiener integrals. In addition the chaos decomposition provides mean and variance formulas for multiple integrals. With the help of these we cast a strong law due to Gladyshev [2] in our language of multiple integrals and prove some extensions of it.

We next study geometric fractional Brownian motion (GFBM), which is the process

$$X(t) = e^{\mu t + \sigma B_H(t)}, \mu \in R, \sigma > 0, t \geq 0.$$

GFBM has been proposed as an empirical model for stock price processes. We show that in a certain sense GFBM satisfies the equation

$$X(t) = 1 + \mu \int_0^t X(s) ds + \sigma \int_0^t X(s) dB_H(s), t \geq 0.$$

Using similar arguments we prove that GFBM does not have an equivalent martingale measure. Then assuming GFBM as the model for stock price we construct a financial strategy which allows riskless gain in the market (*arbitrage* in the language of stochastic finance). This seems to show that GFBM cannot be used as a model for stock prices.

The last question we study is approximation of FBM. A class of approximations that use a particular random walk is provided and is shown to converge weakly to FBM. This approximation is different from the traditional ones which generally use normal random variables.

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